A Hybrid Test for Mathematics; Harry Aarts

Introduction

Nowadays electronic testing packages offer excellent opportunities for students for practicing mathematics.
However, traditionally mathematics exams still consist of merely open exercises.

Question

To what extent can traditional summative mathematics tests be transformed into digital tests, without harming validity and reliability?

Solution

Compose a hybrid test containing both open (pen and paper) and closed (final answer) exercises.

Examples of traditional exercises:

1. [4 pt]
Compute the following integral by reversing the order of integration.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx dy$$

6. [6 pt]
The solution to this exercise must be clearly written down on a separate sheet (including calculations and argumentation)!

The function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by: $f(x,y) = x^2 y$.
Use the method of Lagrange Multipliers to find the maximum value and the minimum value of f subject to $x^2 + 2y^2 = 6$.

Examples of exercises in closed form:

2. [3 pt]
Let $f(x,y) = f(x(u,v), y(u,v))$ and $x(u,v)$ and $y(u,v)$ be differentiable functions.
Use Tables 1 and 2 to determine $\frac{\partial f}{\partial v}(u,v)$ in $(u,v) = (1,2)$.

(x,y)	$(0,0)$	$(1,2)$	$(3,0)$
$f(x,y)$	1	6	-2
$f_x(x,y)$	-1	3	-5
$f_y(x,y)$	-2	8	9

Table 1

(u,v)	$(0,0)$	$(1,2)$	$(3,0)$
$x(u,v)$	-2	3	7
$y(u,v)$	6	0	3
$x_u(u,v)$	1	2	6
$x_v(u,v)$	0	4	1
$y_u(u,v)$	3	-4	2
$y_v(u,v)$	0	5	-3

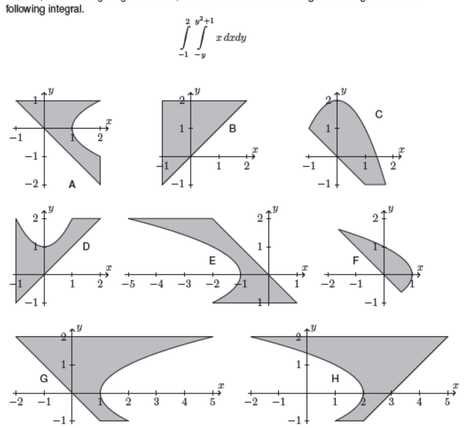
Table 2

Choose from the alternatives below and fill in your answer on the answer sheet:

(a) 31 (b) -21 (c) 0 (d) 25
(e) 58 (f) -32 (g) 52 (h) 26

7. [1 pt]
Choose, from the eight figures below, the correct sketch of the region of integration of the following integral.

$$\int_{-1}^{2} \int_{-y}^{y^2+1} x\,dx dy$$



9. [3 pt]
Consider the following integral.

$$\int_0^4 \int_{\frac{1}{2}x-1}^{1+\sqrt{x}} xy^2\,dy dx$$

The region of integration is depicted in Figure 1.

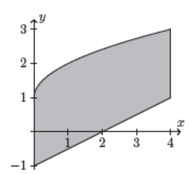


Figure 1: Exercise 9: Region of integration

Change the order of integration of this integral.
Choose the right expression below: either (a) or (b).
Then determine the correct limits of integration and the integrand(s):
either a_1, a_2, a_3, a_4 and $f_1(x,y)$, or $b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4$ and $f_2(x,y)$, $f_3(x,y)$.

(a) $\int_{a_1}^{a_2} \int_{a_3}^{a_4} f_1(x,y)\,dx dy$

(b) $\int_{b_1}^{b_2} \int_{b_3}^{b_4} f_2(x,y)\,dx dy + \int_{c_1}^{c_2} \int_{c_3}^{c_4} f_3(x,y)\,dx dy$

Results:

Pass rate: 76.5 %
(comparable to traditional tests)

Item	Max Score	P	C	α
1	2	0.62	0.259	0.654
2	3	0.26	0.115	0.679
3	2	0.39	0.218	0.657
4	2	0.60	0.195	0.660
5	3	0.78	0.316	0.644
6	6	0.62	0.457	0.614
7	1	0.82	0.257	0.659
8	2	0.87	0.278	0.652
9	3	0.68	0.392	0.630
10	6	0.65	0.542	0.604
11a	1	0.78	0.313	0.655
11b	2	0.63	0.293	0.647
12	3	0.49	0.455	0.625

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Lessons learned:

- No complex exercises in closed form.
- Provide students with sufficient representative practicing material.
- Not all educational targets are suitable to be tested with closed exercises.
- The test package itself might have unexpected technical restrictions as well.

Follow up:

Extension hybrid tests to other mathematics courses.

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