

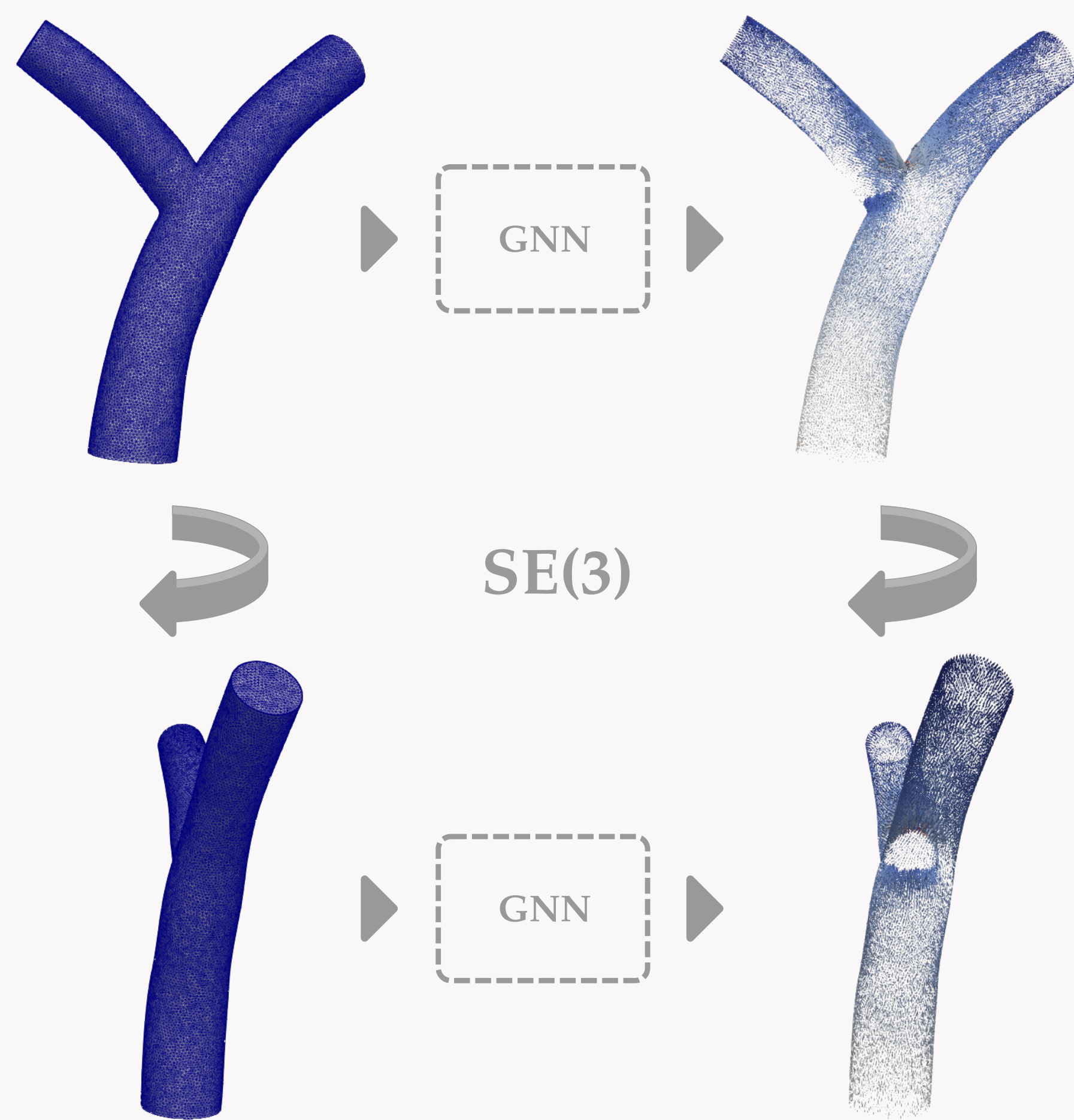
## Symmetry enables data-efficient velocity estimation

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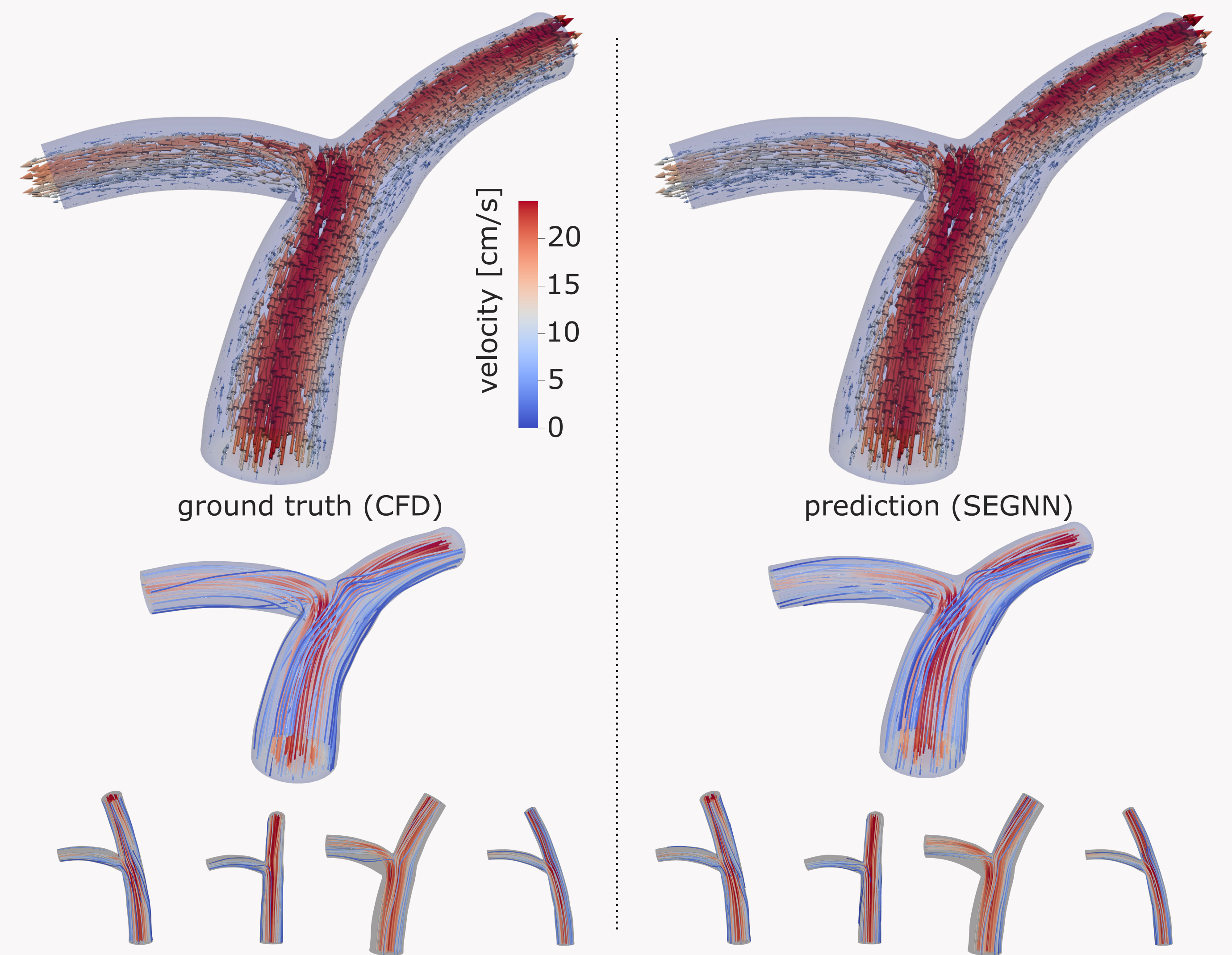
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### Introduction

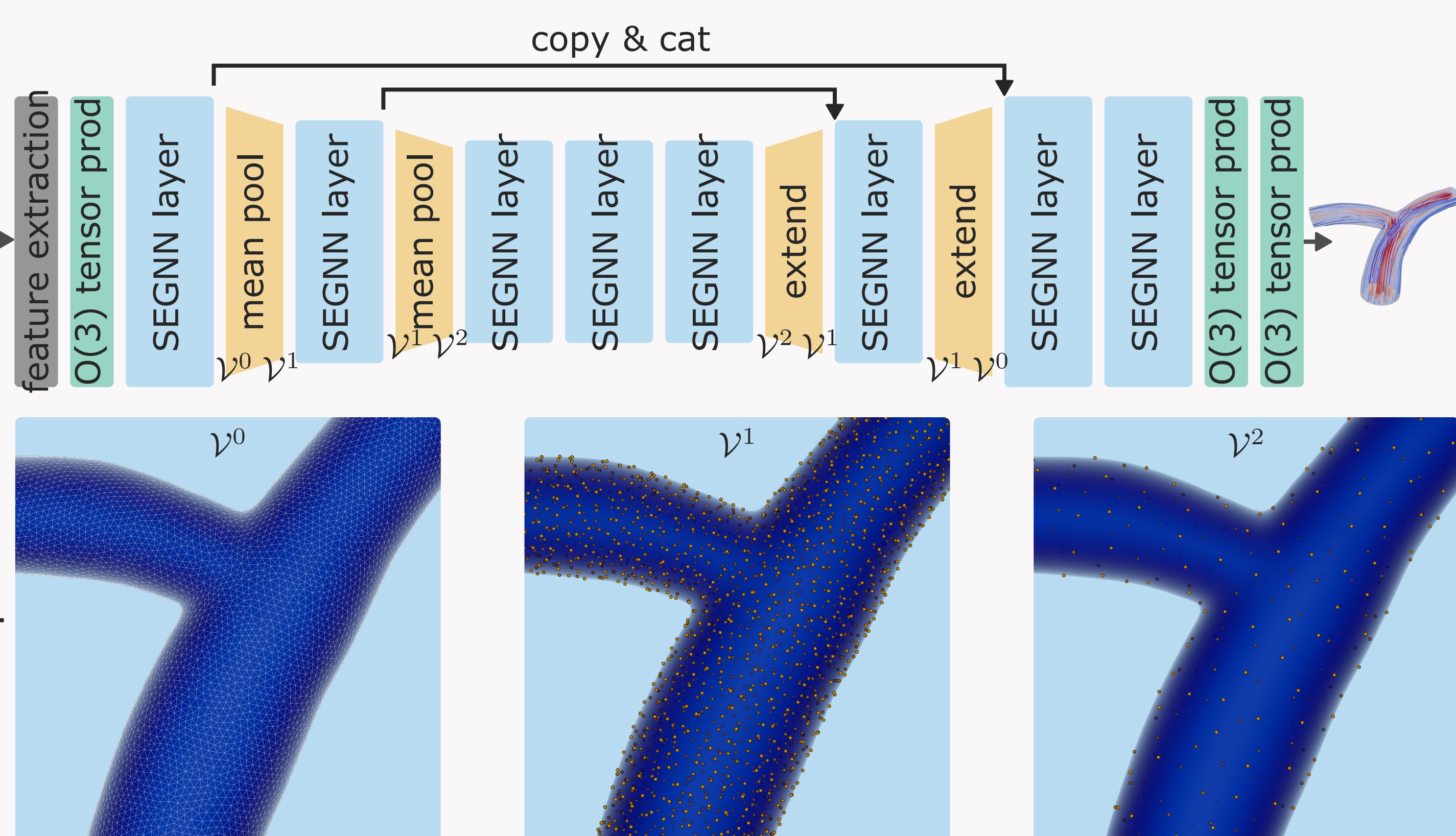
- Hemodynamic velocity fields could be useful in diagnosis, prognosis and treatment planning
- Medical imaging, segmentation and computational fluid dynamics (CFD) to estimate velocity
- CFD is accurate but costly
- Data-driven surrogate methods that learn to infer velocity from artery shape
- Offline training on CFD-generated dataset, fast online inference for new artery
- [Problem] clinical datasets are usually small
- Make use of roto-translational ( $SE(3)$ ) symmetry as inductive bias in graph neural network (GNN)
- GNN can focus on shape-intrinsic properties instead of orientation



### Example results

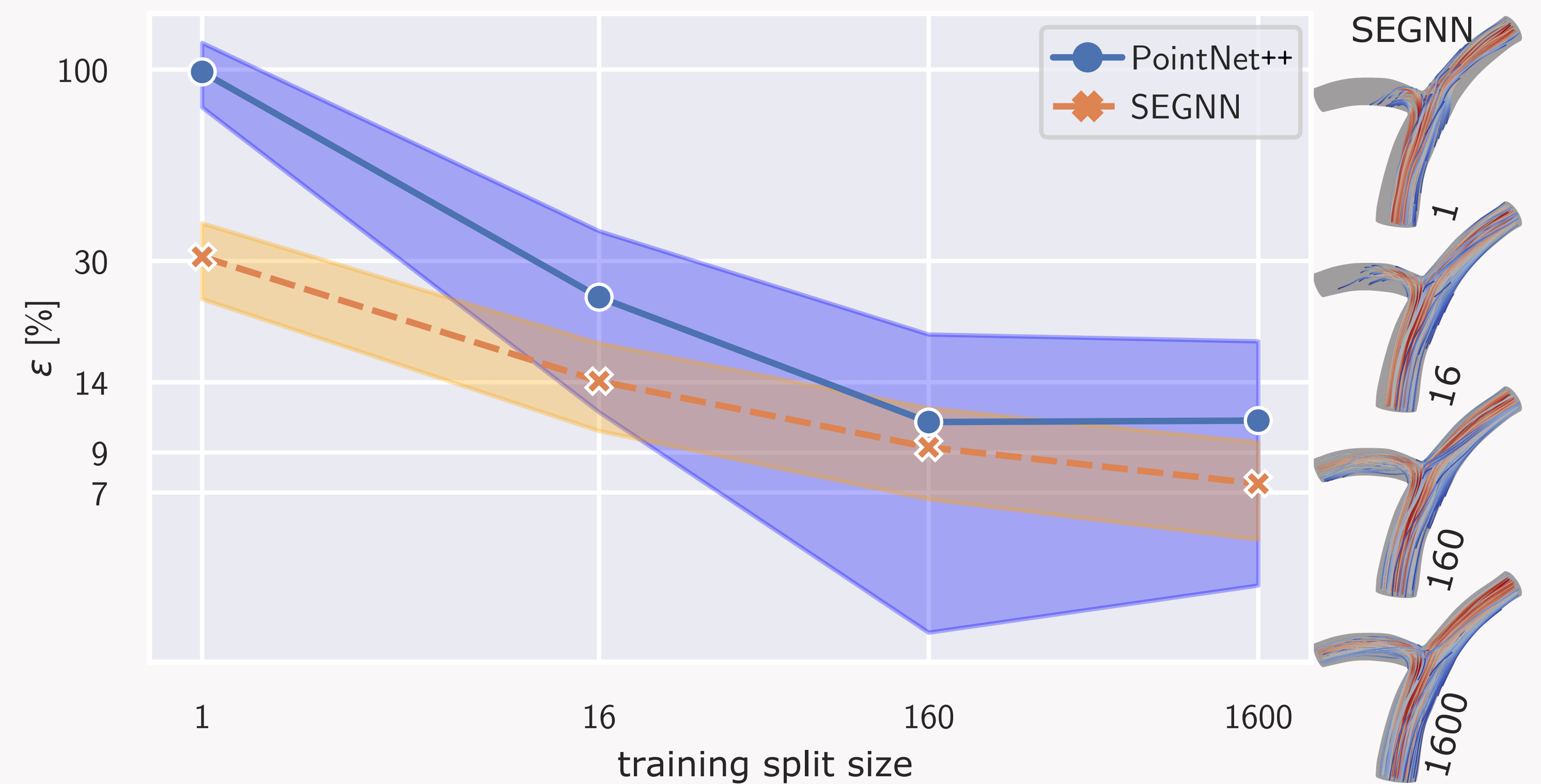


### Neural Network Architecture



- Steerable  $E(3)$ -equivariant graph neural network (SEGNN) [1]
- Three-scale pooling scheme [2]
- Predicts a discrete velocity field mapped to the vertices of the volumetric input mesh
- End-to-end equivariant to  $SE(3)$  transforms (roto-translations)

### Data efficiency



### Conclusion

- We demonstrate how SEGNN can accurately estimate blood velocity in unseen arteries
- We achieve speed-up from 15 min (CFD) to 24.5 s (SEGNN)
- SEGNN is able to learn from a small dataset and makes data augmentation obsolete
- Physics-informed extensions are natural
- We are currently working on pulsatile flow and changing boundary conditions

### References

- [1] Brandstetter, J., Hesselink, R., van der Pol, E., Bekkers, E.J., Welling, M.: Geometric and physical quantities improve  $E(3)$  equivariant message passing. In: Proceedings of the 10th International Conference on Learning Representations (2022)
- [2] Suk, J., de Haan, P., Lippe, P., Brune, C., Wolterink, J.M.: Mesh convolutional neural networks for wall shear stress estimation in 3D artery models. In: MICCAI Workshop on Statistical Atlases and Computational Models of the Heart (2022)

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