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Engineering Students' Initial Use Schemes of ChatGPT as an Instrument for Learning

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Abstract

ChatGPT is a new technological tool with the potential to impact education. Using Vergnaud's notion of "use schemes," we analyzed three interviews with engineering students who discovered ChatGPT and started to develop initial utilization schemes of the tool. Results showed that there were three domains of use of ChatGPT: (a) in mathematics/engineering; (b) for general academic purposes; and (c) in the students' personal lives. Domain (a), with a focus on mathematical modelling, has a relation to mathematics education. Students used ChatGPT to foster their conceptual understanding, to find alternative modelling strategies, to translate mathematical models to computer code, and to optimize this computer code. The students developed a critical attitude in relation to the limitations of the tool and, according to the interview data, their schemes developed over time. The interview data show some evidence for the emergence of a hybrid form of learning in which ChatGPT became a co-agent of learning, an interactive resource with which the students could discuss their ideas. We consider the case as a positive example of how ChatGPT can contribute to student agency in education and support the development of knowledge and student competencies.

Keywords ChatGPT · Instrumental genesis · Schemes of use

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Introduction

ChatGPT is a chatbot based on AI technology which has been trained to provide a detailed textual response to a user prompt or question (OpenAI, 2022). With the release of ChatGPT in November 2022, tools of generative artificial intelligence (GAI) have become easily accessible for the general public, while quickly gaining popularity (Hsu & Ching, 2023a). Soon after, the release of ChatGPT discussions arose about the potential benefits and risks of the use of GAI in education (e.g., Baidoo-Anu & Owusu Ansah, 2023). Bozkurt et al. (2023) suggested that GAI technologies could be used to promote, among others, personalized lifelong learning, flexible learning, improvement of language skills, collaboration and knowledge sharing. At the same time, the authors have warned that GAI comes with several challenges: among others, it may be unreliable as a knowledge source and provide biased information, its uncritical use may impede creativity, and there are privacy and ethics concerns. Moreover, education is essentially a social endeavor in which human agency and teacher–student as well as student–student social interactions play an important role; these social processes cannot be completely replaced by interactions with technology. Finally, students could become overly dependent on GAI technologies, which might have a negative impact on their problem solving and critical thinking skills. Educational authorities and institutes around the world have responded to the rise of GAI and have started giving advice or laying out rules for its use in primary, secondary, and tertiary education (see Hsu & Ching, 2023b, for an overview of the state of affairs towards the second half of 2023).

Students in tertiary education have a plethora of resources at their disposal for their studies, some of which are prescribed or recommended by their institutions and some of which the students find themselves (Pepin & Kock, 2021). They select and use the resources they expect will help them accomplish their goals, for example to obtain high exam grades (Anastasakis et al., 2017).

ChatGPT and other GAI tools are a new kind of resources for the students, because of their innovative ability to generate seemingly "coherent and contextually appropriate text responses through natural language interaction with users" (Hsu & Ching, 2023a, p. 603). In using these resources, students gradually become familiar with the affordances and limitations of the resources, and how they can appropriate them and use them to contribute to their cognitive development: the students gradually develop *utilization schemes* of the resources (Vergnaud, 2009). In the instrumental approach (IA; Rabardel & Bourmaud, 2003), this process is called *instrumental genesis*. In the "Theoretical Framework and Related Studies" section, we elaborate on this process.

Research has been conducted on instrumental genesis taking place when students used digital resources, such as graphical calculators, the CAS software, or dynamic geometry environments, often related to specific tasks (see, for example, Artigue, 2002; Trouche, 2020; Turgut & Drijvers, 2021). However, not much is yet known about the schemes that students start to develop when using GAI as a resource for their studies, with a focus on mathematics. In this article, we set

out to describe elements of these schemes and how they develop, based on data from three interviews with engineering students. The interviews were conducted during the first half of 2023. They were directed to the students' use of ChatGPT in general, and to its use when learning mathematics in particular. Understanding the utilization schemes students develop may provide first insights into the way the affordances and challenges of GAI technologies as identified by Bozkurt et al. (2023) play out in practice, and suggest student guidance that may be beneficial.

In this explorative study, we therefore answer the research question:

Which utilization schemes of ChatGPT have students started to develop about the ways in which ChatGPT can contribute to their learning experiences, as particularly related to mathematics?

After the "Introduction" section, we explain the theoretical framework we have used and related studies on the use of GAI in education. We then describe the methods of data collection and analysis in the "Methodology" section. In the subsequent sections, the findings and conclusions are presented.

Theoretical Framework and Related Studies

The Instrumental Approach

We draw on the instrumental approach (Rabardel & Bourmaud, 2003; Trouche, 2004) to analyze how students have developed their use of ChatGPT. The instrumental approach was introduced to mathematics education in an effort to understand how the introduction of digital technologies in the mathematics classroom affected student activities and student learning (Sinclair et al., 2022). According to this approach, human activities are mediated by artifacts such as technological tools (Verillon & Rabardel, 1995): the students learn how to use the artifact to accomplish their goals, while the characteristics of the artifact influence the cognitive development of the student.

Thus, the instrumental approach characterizes the interaction with an artifact (or group of artifacts) as the conjunction of two processes: instrumentation and instrumentalization. Both processes explain how a user appropriates an artifact, produced in a cultural context, and how this artifact subsequently develops as a means to an end, an instrument for this user (Verillon & Rabardel, 1995). Instrumentation is the name of process, in which the affordances and constraints of tools influence the user's practice and knowledge. Instrumentalization is the name of the process, in which users adapt the tools to their own needs (see Fig. 1). According to Trouche (2004), "the instrumentalization process can go through different stages: a stage of discovery and selection of the relevant functions, a stage of personalization (one fits the artifact to one's hand) and a stage of transformation of the artifact, sometimes in directions unplanned by the designer" (p. 293). The combination of the two processes is called *instrumental genesis*, the development of an artifact into an instrument. What distinguishes an instrument

from an artifact is the knowledge users develop about the artifact during the instrumental genesis, in the form of a dynamic scheme of use (Vergnaud, 2009). Such a scheme consists of four interconnected aspects:

- An intentional aspect: the aims of the artifact, that is, the goals for which it can be used, related to the different situations in which its use is considered appropriate.
- A generative aspect: the rules regarding the use of the artifact, that allow the user to accomplish the goals.
- An epistemic aspect: this aspect consists of concepts-in-action (concepts considered relevant) and theorems-in-action (declarative statements, held to be true about the use of the artifact); the concepts and theorems are developed *in-action*, that is, while using the artifact. They are not necessarily correct or final, and users may adapt and extend them with increasing experience.
- A computational aspect: the computational aspect refers to the possibilities of 'inference' which are used to generate goals and rules, to infer properties, and to arrive at (new) concepts and theorems.

The process of instrumental genesis including the formation of a scheme is schematically shown in Fig. 1.

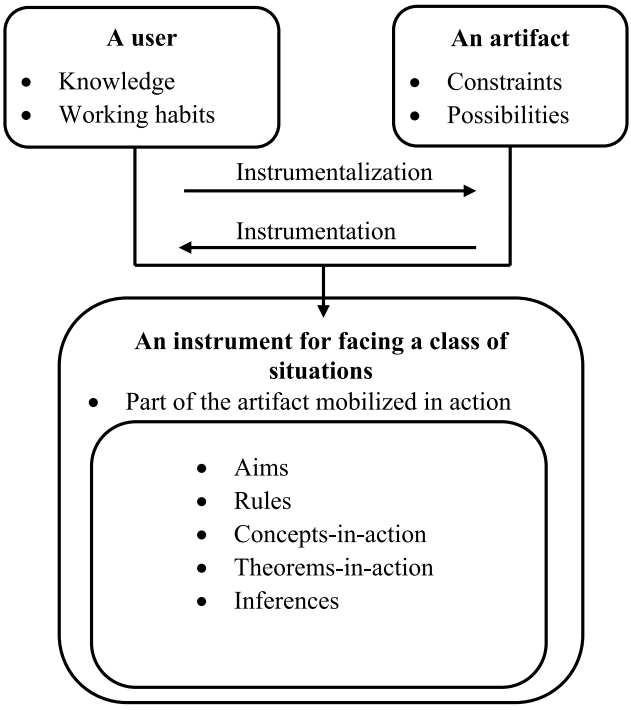

Fig. 1 A representation of instrumental genesis (adapted from Trouche, 2020)

Our research question focuses on the aspects of the schemes that students develop, shown in Fig. 1. Although it is not the focus of the research question, it is useful to note that we think of ChatGPT in terms of a digital resource for the students, among the many resources that students have at their disposal during their studies of mathematics (Anastasakis et al., 2017; Pepin & Kock, 2021). We take the notion of a resource as anything likely to resource the students' mathematical practice (Gueudet & Pepin, 2018). The different kinds of resources can be categorized as "material" (digital or non-digital) curriculum resources (resources prescribed or recommended for a particular course or learning goal, e.g., textbooks; worksheets); "non-material" resources (e.g., social resources, such as conversations with supervisors, peers, and friends); cognitive resources (e.g., concepts and techniques); general non-curricular (digital) resources (found by the students themselves, e.g., Wikipedia) (Pepin & Kock, 2021; Pepin et al., 2024a). The different kinds of resources have the ability to exercise forms of co-agency during student work, that is, to interactively shape students' actions and decisions (Pepin et al., 2024b; Salinas-Hernández et al., 2022). At the same time, students exercise agency when selecting the resources that they intend to use and decide how to use them (Pepin & Kock, 2021).

Related Studies

As the introduction of ChatGPT (and other GAI applications) in education took place approximately 2 years prior to the time of this publication, the number of studies on its use in mathematics education is still limited. This applies even more for studies from the perspective of the instrumental approach: a search in the ERIC database did not show any results for studies from that perspective on the use of AI or GAI applications by students.

To provide a background for this study, we briefly discuss selected studies on (a) AI tools before the emergence of GAI, in particular what the tools were used for; (b) examples of the use of a GAI application, ChatGPT, in tertiary mathematics and physics education; and (c) how the relation between students and AI and GAI applications can be characterized.

AI Tools Before the Emergence of GAI

AI tools with a narrower focus have existed for approximately 30 years (Zawacki-Richter et al., 2019). Zawacki-Richter et al. conducted a systematic review of 146 studies published between 2007 and 2018 on the use of AI in higher education. They found that most studies were conducted in the field of Computer Science and STEM, using a variety of approaches and applications. The authors identified intelligent tutoring systems (ITSs) as important AI applications in the studies. ITSs are AI applications used to teach course content. Depending on the application, they may be adaptable and allow for personalization, that is they are able to monitor and guide students, provide personalized feedback, adapt learning materials to students' needs (or preferences), and facilitate collaboration between learners. In contrast to the more general GAI applications, ITSs usually contain models of the student, the teacher, as well as domain-specific knowledge,

and are able to detect and evaluate errors. In terms of limitations of AI applications, Zawacki-Richter et al. (2019) identified potential risks (such as student misconceptions regarding the nature of AI), and ethical and privacy implications. They remarked that the studies mostly lacked a critical reflection on these limitations.

A dialog in natural language between the student and the computer has been the basis of several ITSs. Examples are a natural language tutorial system for introductory college Physics (Chi et al., 2011), a tutoring system that teaches natural deduction to undergraduate students and provides support at different levels (Miwa et al., 2014), and a system to support students in parametrized modelling activities (Rojano & García-Campos, 2017). However, in these systems, the natural language capabilities of the support system were restricted, demonstrated by the fact that the system did not recognize several student answers. It seems that such limitations have, at least partly, been overcome in GAI applications such as ChatGPT.

Examples of ChatGPT in Tertiary Mathematics and Physics Education

The first studies have been published on the use of ChatGPT to assist learning in mathematics and the sciences. In one study (Barana et al., 2023), 40 Italian undergraduate students were asked to solve problems in combinatorics, with the help of ChatGPT if they wanted. The purpose was to find out the problem-solving and critical-thinking strategies the students would use. The results showed that the students used ChatGPT to find new and different ways to solve the problems and test the solutions. They checked ChatGPT's answers and used ChatGPT to check their own answers. However, it was noticed that not all students used the assistance of ChatGPT. The authors concluded that they saw potential benefits in its use, and potential harm (e.g., due to misinformation) to student learning was limited.

In another study (Ding et al., 2023), the research team administered an examination assignment on light and radioactivity to a group of 40 students of an introductory physics class at a public university in the USA. The students could use the assignment to regain lost credits on a regular examination. They were allowed to use/chat with ChatGPT to complete the assignment and were asked to complete a survey regarding their experiences. The results showed that many students blindly trusted the answers generated by ChatGPT. Moreover, those were students who often considered ChatGPT as easy to use and reported that they were likely to use it in the future. The authors identified several misconceptions regarding ChatGPT (e.g., that it is infallible and possesses human-like traits) and recommended teaching "AI literacy" to students, in order to maximize its potential benefits in education. It is not clear why these studies show somewhat different results. However, they do point to the value of understanding the utilization schemes students create when using ChatGPT.

Relation Between Students and AI or GAI Applications

We describe two ways to characterize the relation between students and AI or GAI applications, which are informative in understanding the student utilization schemes in this study.

First, based on a literature review, Ouyang and Jiao (2021) have claimed that three consecutive paradigms have characterized the use of AI applications: (a) AI-directed (learners as recipients: AI represents knowledge models and directs cognitive learning); (b) AI-supported (learners work as collaborators with AI); and (c) AI-empowered (learners are in the lead and take agency to learn with AI). The paradigm characterizing the particular use of an application does depend not only on the application itself but also on the context and the specific educational context in which it is used. The two example studies on the use of ChatGPT left the students with considerable agency regarding the use of ChatGPT and how to use it, and are thus characterized by the AI-empowered paradigm.

Second, Lodge et al. (2023) described the relation between the learner and ChatGPT in two dimensions along two axes: (1) an individual—collective axis (emphasizing the individual learner or a collaboration between human and computer) and (2) an offloading—extending axis (reducing student cognitive load or enabling new tasks and possibilities). Combinations give rise to four approaches to the use of AI in education: individual + offloading (similar to a calculator); individual + extending (e.g., by enhancing human creativity); collective + offloading (e.g., asking questions; "GAI as a coach"); collective + extending ("GAI as a teammate"). In the latter three approaches, GAI systems work in conjunction with human learners with the possibility to support both cognitive and metacognitive aspects of learning. This is referred to as hybrid learning (Molenaar, 2022), conceived by Lodge et al. (2023) as: "where generative AI becomes a prosthesis for the social aspects of learning in the way that the extended [individual] mind describes a prosthesis for the individual aspects of learning" (p. 123). GAI thus becomes a co-agent in the students' learning process (e.g., Pepin et al., 2024b; Salinas-Hernández et al., 2022).

The three paradigms describe where the agency is located in the learning process: is it mostly with the application, is it shared or is it mostly with the student? The two dimensions emphasize how students use GAI applications: to make their work easier (offloading) or to open up new possibilities.

Methodology

The data used in this study was collected during three semi-structured student interviews. The first interview was not directed at our research question, but ChatGPT was unexpectedly mentioned as an important resource for the interviewees and we asked additional questions on the use of this resource. In the analysis phase, we noticed aspects of scheme formation in the students' responses. We conducted the additional two interviews, with the purpose to collect additional data on the students' formation of utilization schemes of ChatGPT. All three authors were present during interview 1 and interview 2; interview 3 was administered by the first author. Below, we present an overview of the three interviews. Sample questions are provided in Appendix A.

Interview 1: a group interview conducted with four students as part of a study on student learning experiences at a university of technology in February 2023

(Kock et al., 2023). The students followed bachelor programs of electrical and mechanical engineering and in applied physics. They were involved in a multidisciplinary course on *Sociophysics*, the application of modelling techniques from mathematics and physics to describe the behavior of humans in crowds. This course was set up following principles of challenge-based education (see Gallagher & Savage, 2020): students are given or select a general challenge with societal relevance from which they derive a specific problem they want to address and questions they want to answer. They then use inquiry and design processes to arrive at answers to their questions and often a prototype solution to their problem (for example, a mathematical model).

Guidance and structure are provided by the course organizers. It is expected that students develop disciplinary knowledge along with professional competences while they work on a challenge that is motivating, meaningful and addresses the solution of problems in society. In this particular second-year course, student groups selected the challenge to optimize the efficiency of a train platform boarding process. One of the aims of the interview was to investigate which resources the students had used and how these had contributed to their learning. During the interview, the students mentioned that they had "discovered" ChatGPT as a resource and that it had become important for their work and learning. We used the interview to find out how the students had developed elements of a scheme regarding the use of ChatGPT. Three of the four students (s1, s2, and s3) participated in the interview section on ChatGPT.

Interview 2: the students who participated in interview 1 were invited for a follow-up interview in May 2023, to concentrate on their further use of ChatGPT. One student responded and brought his friend (both students of electrical engineering), who had not participated in interview 1. We allowed the friend to participate as the two students made clear that they had largely developed their use of ChatGPT together. The purpose of this interview was to find out how the students' use of ChatGPT and the scheme formation had developed.
Interview 3: to provide a broader basis for our analysis, engineering students from a university of applied sciences were asked to participate in an interview (conducted in June 2023) if they were regular users of ChatGPT. One student volunteered, in his second year of a Bachelor program in Mechatronics. The purpose of this interview was to obtain a first comparison regarding the use of ChatGPT and associated scheme formation from a different tertiary educational setting. This interview was conducted in Dutch and quotations presented in the article were translated from Dutch by the first author.

The interviews were recorded and subsequently transcribed, using automatic transcription (auto-transcription function of Microsoft Teams), after which the transcription files were manually corrected by the authors using the original audio files. The transcriptions were qualitatively analyzed using a deductive qualitative data analysis process (Bingham, 2023). Based on the instrumental approach, we used Vergnaud's (2009) aspects of schemes as categories to code the student responses regarding the

use of ChatGPT. However, instead of Vergnaud's aspect of "inferences," in interviews 2 and 3 we used the related theme *development over time* based on student responses about their evolving scheme formation (see Table 1 for the themes and sample quotations).

The inferences category from Vergnaud's framework was adapted in this study due to the nature of the interviews we conducted. While "inferences" play a role in the analysis of instrumental genesis, we found that during the interviews in this specific context, students were not explicit about inferences regarding their actions or knowledge. Instead, they were explicit about the ways in which their understanding and use of ChatGPT evolved over time. Therefore, we considered it more relevant to analyze how students' use schemes developed and adjusted over time, as captured in the "development over time" category. This adjustment allowed us to reflect the dynamic nature of the students' changing schemes, as an outcome of their inferences, rather than focusing on possible implicit inference mechanisms themselves.

During interviews 1 and 2, the participating students responded or added to each other's comments. For example, on many occasions during interview 2, one student confirmed the other student's statements by nodding or non-verbal sounds (such as "mm hmm"). This contributed to the richness of the interview data, but made it difficult to identify individual student scheme formation. Therefore, we categorized the student responses for each interview instead of for each student separately. Comparing the results from the different interviews in the different themes allowed us to identify draw conclusions on the development of utilization schemes of ChatGPT. In the analysis, the focus was on the (developing) schemes of the students. Although interviews 1 and 2 were group interviews and interview 3 was an individual interview, a comparison regarding scheme formation was possible based on the content of the student utterances. The inclusion of interview 3 was valuable as it broadened the range of students' experiences captured by the interviews: it added the perspective of a student from engineering education at a university of applied sciences. The analysis was conducted by the first author and discussed in the research team until a consensus interpretation was reached.

Results

We present a summary of results for each interview. Details with summaries of student quotation categorized according to Table 1 can be found in Appendix B. Where necessary for clarity, contributions by the different students are indicated in line with Appendix B (e.g., s1 for student 1). After the summaries, we present the themes regarding scheme formation emerging from a comparison of the three interviews.

Interview 1

The students said that they had learned informally about the release of ChatGPT and had started using it (s3: only few days after its release), first in a playful way, then more seriously for their project. At the time of the interview, the students had started

Table 1 Categories for qualitative data analysis based on Vergnaud (2009)

Category	Description	Sample quotation
Situations	Reference to the situation or context in which ChatGPT was used	"During the course Applied Mathematical Algorithms"
Aims	Reference to the purpose for which ChatGPT was used	"It's a great tool for us to enhance our learning process, (…) when we didn't understand certain concepts"
Rules	Reference referred to the ways in which ChatGPT was or should be used successfully	"Use the main results [of ChatGPT] and then investigate further; don't use it for final things"
Concepts-in-action	Reference to relevant concepts that describe the characteristics of ChatGPT	"It is like Googling and getting the main ideas"
Theorems-in-action	Statements considered to be true regarding (the use of) ChatGPT	"Most details are wrong"
Development over time	Statements of how a student's use and understanding of ChatGPT has changed over time	"After using it for months, you start to really get the (feeling) on what things to believe and what things not to believe"

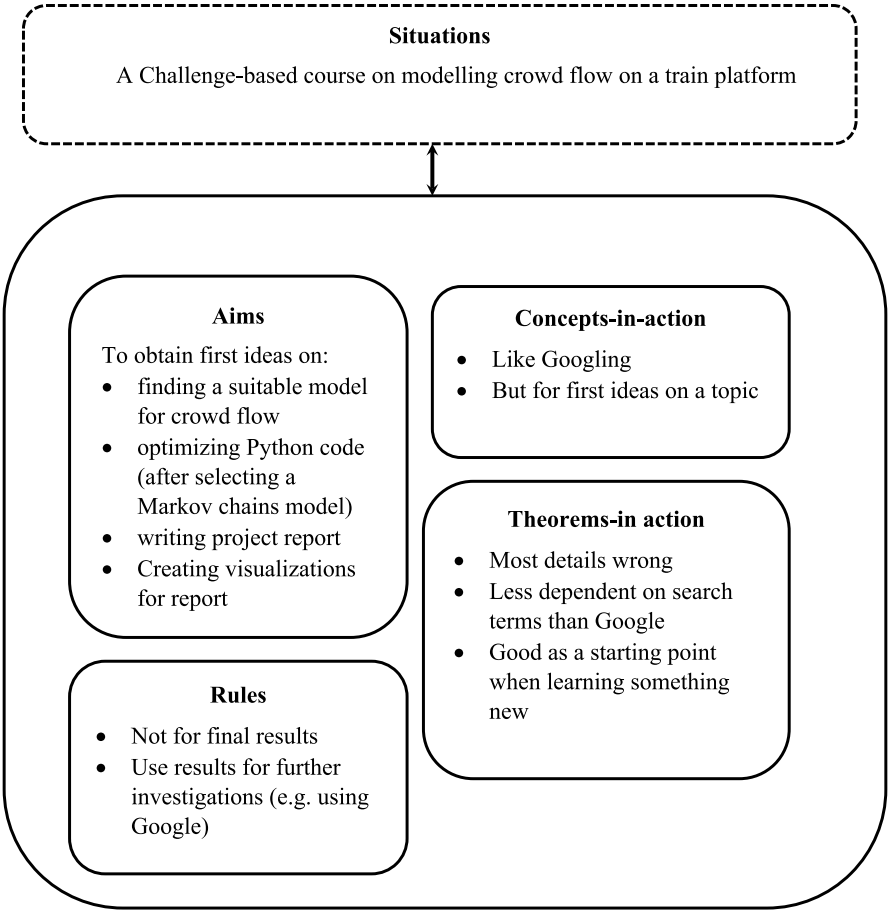

Fig. 2 Utilization scheme of ChatGPT interpreted from interview 1

to develop schemes regarding the use of ChatGPT. In Fig. 2, we present the scheme that we interpreted from the interview data in Table 3.

The students had started to use ChatGPT for mathematics-related tasks in their project: to find and compare different approaches to model (simulate) crowd flow on a train platform. Once they had decided to use a method based on Markov chains and were working on the implementation, they used ChatGPT to find ways to optimize the Python code. Later, when working on the project report, they used ChatGPT to generate texts for its introduction and to get ideas on how to visualize their simulation results. The students were exploring the possibilities of ChatGPT to help them accomplish their goals in the project. After they had experienced its usefulness in one situation, it was "nice to like see that it could be useful for other stuff."

The students were aware of the limitations of ChatGPT, as they found that "most details are wrong." This is reflected in the concepts-in-action and theorems-in-action

we derived from their responses, and plays out in the aims and rules of their schemes. Based on the first ideas generated by ChatGPT, further investigations were necessary, for example using traditional search engines like Google. And Python code generated by ChatGPT had to be checked and corrected manually.

Conceptually, the students considered ChatGPT as a more efficient search engine than Google, because it could provide useful answers to questions, even if the students did not know precise and suitable search terms.

Interview 2

At the time of interview 2, student s3 was using the paid version of ChatGPT, because of the time it saved him, as well as the access to the latest version and to several plugins (e.g., the browsing mode and the connection with Wolfram Alpha). Student s4 was using the "free" version, which she had been using almost since its release: "I am part of a student team about AI, so I follow the news about it. I was aware that it was coming out and have been using it for a long time." Both students said they daily used ChatGPT, for their studies and for private use, except during (holiday) periods when they were not studying. Figure 3 shows the scheme that we interpreted from the interview data in Table 3, with a focus on its use for their studies in electrical engineering.

As the aims show in Fig. 3, they used the ChatGPT to carry out tasks more efficiently, and to foster their learning (e.g., to understand better the concept of "amplitude modulation"; s4). Some tasks involved forms of mathematical modelling and programming. However, they hardly used ChatGPT to solve mathematical problems directly, as it "makes a lot of mistakes" (s4). The students said they used ChatGPT as a tool to study, but not as a tool to do their work for them. As student 3 explained:

> You can think of it as having a discussion with someone that has read every book of mathematics for you to actually understand. Once you understand it then you can be confident that your answer is correct. So instead of seeing it as something that will do the homework for you, you can see it as something that if you tell it to do your homework, it will get it wrong, but it can help you understand things to get it right.

Student 3 had also used ChatGPT for what he called "career guidance," to investigate what Master program he would follow after finishing his Bachelor program.

In terms of development over time, after using ChatGPT for some months, students 3 and 4 said they had become much more proficient in its use: they had acquired "the feeling on what to believe and when it is lying" (s4); they had learnt to give better prompts and therefore to use it for more complicated tasks (e.g., in programming; "I can give it context and more information and I know how to give that information; I have a broader range of things I can use it

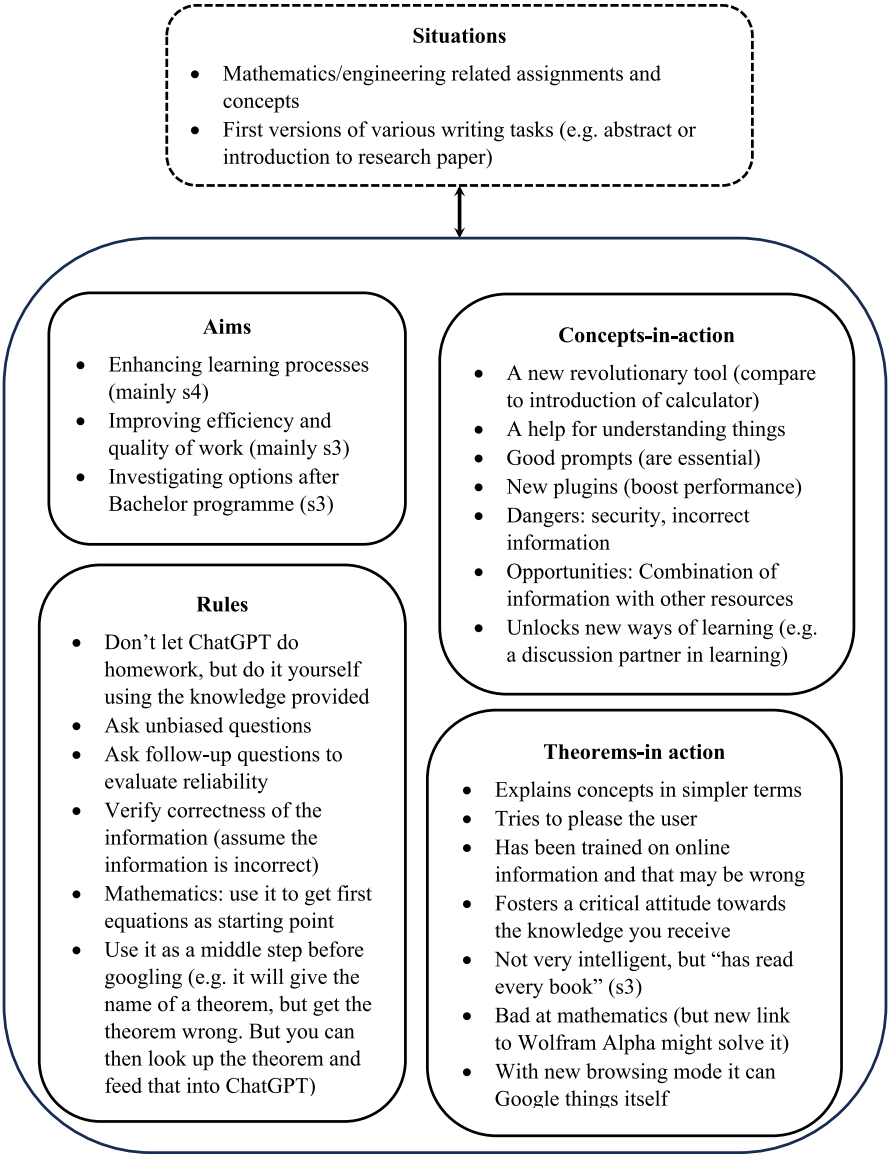

Fig. 3 Utilization scheme of ChatGPT interpreted from interview 2

for" – s3). The students expected that there was still more to explore, especially as new plugins became available. In some courses, the lecturers had encouraged them to use ChatGPT: "in Electromagnetics 2 (and other courses), the teacher said everyone's going to use ChatGPT, so you might as well use it to learn how to use it. And then, it is important obviously to mention that you use it" (s4).

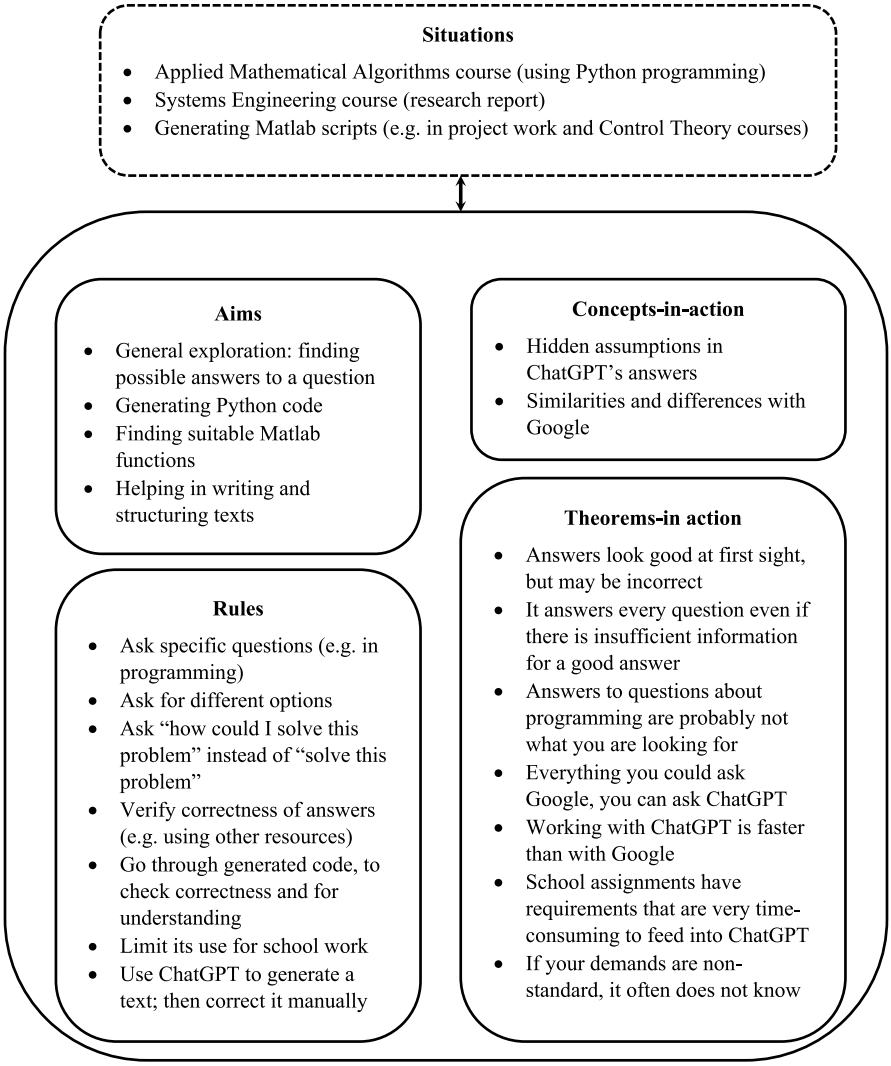

Fig. 4 Utilization scheme of ChatGPT interpreted from interview 3

Interview 3

Student 5 said he had found out about ChatGPT by means of technological news he followed on the internet. He had started using it since March 2023 and used it two or three times a week now. Figure 4 shows the scheme that we interpreted from the interview data in Table 3, with a focus on its use for student 5's studies in mechatronics.

Student 5 used ChatGPT in an exploratory way to find possible (that is, not final) answers to questions, and to help him write texts. The situations in which he used it

were related to his personal life and to his studies, although he tried to minimize it for school work ("because I am at school to learn things"). As school related examples, he mentioned Python code generation (in an Applied Mathematical Algorithms course), finding functions in MATLAB and creating a report (in a Systems Engineering course). In the Mathematical Algorithms course and the Systems Engineering course, the course lecturers were expecting the use of ChatGPT by the students. He had not tried to use ChatGPT to carry out mathematical calculations, but he had used it to explain mathematical concepts (e.g., the "travelling salesman problem") when he had not paid attention during the lesson.

In terms of his scheme development over time, student 5 had found out that it is necessary to ask specific questions, because otherwise ChatGPT would "make its own assumptions." This he considered a limitation of the tool and he had changed his approach: in programming he only used ChatGPT to find answers to "small" specific questions preferably related to a standardized problem (e.g., "how to write this string to a file"). In his view, ChatGPT was often faster and more to the point than Google in providing the information he was looking for.

He thought it would be useful for him to learn to work with AI tools. They would probably be used a lot in his future professional life, because "making things with AI will be cheaper than having people do it."

Comparison of the Interview Results

We have compared the interviews results and schemes in terms of the following: (a) the situations in and aims for which the students have used ChatGPT; (b) evidence of instrumentation and instrumentalization processes; (c) ChatGPT as a resource for the students; and (d) hybrid learning with ChatGPT.

Situations and Aims

The schemes showed some agreements regarding the *situations* in which the students reported to have used ChatGPT for their studies:

(a) programming (in Python or MATLAB) related to mathematical modelling in mathematics and engineering;
(b) situations during their studies in which they were looking for explanations of concepts (amplitude modulation; travelling salesman problem) or were looking for (mathematical) methods to solve a problem (ways to model crowd flow);
(c) writing project-related or course-related texts (related to academic skills).

The students also reported the *aims* for which they used ChatGPT. In the first interview, the student group was still exploring the possibilities offered by ChatGPT and found it could be used to "generate main ideas," because "all the details are wrong." This appears similar to the aims reported by student 5 in interview 3, who, at the time of the interview, had been using ChatGPT for approximately 3 months.

However, in the second interview, the students reported to use ChatGPT almost daily to enhance their learning processes, to study more efficiently and to produce higher quality work. Both students had been using ChatGPT for more than 6 months and it appears that its use had become part of their routines, while they were also still exploring new possibilities.

Instrumentation and Instrumentalization

The interviews and schemes show evidence of instrumentation processes (the affordances and constraints of the resource have an impact on the student activities). In terms of affordances, the students considered ChatGPT as a more flexible and sometimes "faster" form of Google (concept-in-action; interviews 1 and 3) that could provide useful information, even if questions were not precise. They used it to explore concepts, to get new ideas for their projects and to generate computer code. Students s3 and s4 used it as a discussion partner, to understand (mathematical and engineering) concepts better, thus unlocking "new ways of learning." In terms of constraints, theorems-in-action in the student schemes showed awareness that the information provided by ChatGPT could be unreliable or not directly useful (e.g., "tries to please the user").

The students adapted their aims to these limitations and mentioned rules in their schemes to deal with them: checking the information provided by ChatGPT using other resources; checking if programming code would run and give the desired results; processing the results provided by ChatGPT (e.g., an email of a report); using unbiased prompts or restricting the type of questions asked (specific enough, asking for options, rather than a single solution). The students in interviews 1 and 3 said they used ChatGPT mainly for exploration (interview 1: "getting first ideas").

The interviews and schemes also show evidence of instrumentalization processes (how students adapted the use of ChatGPT to their own needs). According to the students, a key element in using ChatGPT was to use the right prompts to obtain useful information. In interview 2, students 3 and 4 said they had gradually learnt to give better prompts or use a multistep approach ("the middle step before Googling"), to extend the range of situations in which they could use ChatGPT (e.g., for more sophisticated problems, among others by providing more context). Also, student s5 said his way of prompting ChatGPT had improved over time, but he didn't want to use it too much for his studies as he was "at school [university] to learn things."

The instrumentation and instrumentalization processes were still continuing at the time of the interviews. This was partly due to the development of ChatGPT itself. Students 3 and 4 expected their use of the ChatGPT (and hence their schemes) to change when they gained more experience using recent plugins (e.g., the connection to Wolfram Alpha providing better access to mathematical content); they expected that there would be much more to explore in the future. Student 5 expected that tools like ChatGPT would become widely used in the (engineering) workplace; this was one of the motivations for him to get familiar with it.

ChatGPT as a Resource

For the students who participated in interviews 1 and 2, ChatGPT had been a digital non-curricular resource: the students had found it and had started to use it by themselves, because they thought it useful for them during their studies. For student 5 in interview 3, ChatGPT had been both a non-curricular and a curriculum resource: in several situations, student 5 had used it on his own initiative, but in two courses teachers had expected the students to use it for particular tasks (programming of mathematical algorithms and structuring/writing texts). According to student 5 the mathematics teacher had encouraged the students to use ChatGPT, so that they could focus on mathematical understanding rather than on details of writing Python code. Some teachers of students 3 and 4 had started to encourage the use of ChatGPT in general, as they expected it to be a resource everybody would be using in the future (similar to the introduction of the calculator).

The students described ChatGPT as an application that changed their mathematical and engineering practice: to find mathematical modelling methods, to obtain conceptual explanations, to explore new ideas, as a discussion partner when studying, to generate or improve computer code, to structure and write first versions of a text. Often, it worked best when combined with other resources. Students 3 and 4 even called it "revolutionary."

Hybrid Learning with ChatGPT

The interviews show evidence that ChatGPT was used by the students to support cognitive aspects of their learning, for example in programming, in exploring mathematical modelling approaches and in coming to understand domain-specific concepts. We did not see evidence that ChatGPT directly supported metacognitive aspects of student learning. However, indirectly, the ChatGPT responses triggered the students to regulate their learning process. For example, the students did not use ChatGPT to solve mathematical problems directly as, according to students 3 and 4, it was "bad at mathematics." Instead, they used ChatGPT to help them study, for example by "having a discussion with someone who has read every book of mathematics" not with the purpose to let ChatGPT do the work for them but to "understand things to get it right." Moreover, all students were aware of the possibility that ChatGPT would hallucinate or give incorrect information and critically considered the information they received. Hence, ChatGPT appeared to provide students with opportunities to reflect on their thinking process, promoting meta-cognitive awareness.

Conclusions and Discussion

In this exploratory study, based on three interviews in which five different students participated, we set out to answer the research question: *Which utilization schemes of ChatGPT have students started to develop about the ways in which ChatGPT can contribute to their learning experiences, as particularly related to mathematics?*

In the analysis, we interpreted the student responses from the three interviews as utilization schemes in line with Vergnaud (2009), showing that ChatGPT had developed from an artifact to an instrument for the students (Verillon & Rabardel, 1995). The students mentioned situations in which they had used ChatGPT, aims for which they had used it and rules on how it could be successfully used. The concepts-in-action and theorems-in-action which described their cognitions about ChatGPT were consistent with the situations, aims and rules. The scheme interpreted from interview 1 shows that the students only recently started to use ChatGPT and were still exploring its use (with only a few concepts-in-action, theorems-in-action and rules).

However, ChatGPT contributed to the students' work to develop a mathematical model of crowd flow on a train platform. It made their work easier and allowed them to explore possibilities that would be more difficult to explore without ChatGPT. The scheme interpreted from interview 2 was more sophisticated: it contained more complicated situations of use, more rules to use the tool (stressing the importance of prompts) and more theorems-in-action. The students had been using ChatGPT routinely to study more effectively and efficiently. The scheme interpreted from interview 3 contained similarities (e.g., its use for programming tasks), but also differences compare to scheme from interview 2: student 5 had limited the use of ChatGPT for study purposes and, regarding programming, it appears he had simplified the range of questions he asked ChatGPT, while students 3 and 4 had extended this range.

In line with Trouche (2004), the students went through a stage of discovery and selection of the relevant functions, and then entered a stage of personalization, in which they found ways of using the tool that worked for them. Student 3 described how he included ChatGPT in a multistep approach to arrive at the answers he needed. This might be considered a transformation of the artifact, possibly in a direction unplanned by the designer (Trouche, 2004). The process of instrumental genesis has not yet ended, partly because ChatGPT is still developing, for example by the inclusion of new plugins. Students 3 and 4 were aware of this and were looking forward to these new developments.

The schemes showed three domains of use of ChatGPT: (a) in mathematics/engineering; (b) for general academic purposes; and (c) in their personal lives (not considered in the schemes). Domain (a), with a focus on mathematical modelling, has a relation to mathematics education. Students used ChatGPT to foster their conceptual understanding, to find alternative modelling strategies, to translate mathematical models to computer code and to optimize this computer code. The students were aware that often additional steps needed to be taken to make the results meaningful and usable.

The interviews show evidence that the students used ChatGPT in line with the individual approaches to GAI in education identified by Lodge et al. (2023): individual-offloading (e.g., asking for Python code) and individual-extending (e.g., exploring possible MATLAB functions; asking for conceptual explanations; using ChatGPT as a discussion partner). There is some evidence for the two collective approaches, for example where the responses from interview 1 suggest that the student team at times used the collective-extending approach ("GAI as a teammate," e.g., for exploring and then discussing mathematical models). It can be argued that here we have observed an emerging hybrid relationship between the students and technology, in the sense that the effects extended beyond what could be provided

by a peer or teacher (Lodge et al., 2023). On the occasions in which ChatGPT has taken the role of a teammate or an interactive resource with which the students could discuss their ideas, it has become a co-agent, in the learning process. This is in line with the agentic nature of non-human resources found during an earlier study (Pepin et al., 2024b; Salinas-Hernández et al., 2022).

The emphasis regarding agency was with the students themselves. They used ChatGPT according to Ouyang and Jiao's (2021) AI-empowered paradigm: they decided when to use ChatGPT as a resource, what to use it for, and how to do that. It can be claimed that ChatGPT has the potential to give students more agency over their learning and, in certain cases, and, if well used, makes them more independent of their teachers. In part, this may be caused by the fact that the ChatGPT gives the students access to new resources, that they could not have easily accessed themselves, even with the help of a search engine.

All students showed awareness of the constraints of ChatGPT and in particular of "hallucinations" and possible unreliability, unlike the students in the study by Ding et al. (2023), some of whom considered ChatGPT infallible and possessing human-like traits. This result may be due to the selection of students who participated in our study: they were volunteers to be interviewed and followed the news on technological and AI developments, which may have contributed to their knowledgeable and critical attitude regarding the constraints of ChatGPT. Notably, students 3, 4, and 5 mentioned an uncritical attitude towards ChatGPT as a potential risk for other students.

Limitations of the Study

This exploratory interview study has a number of limitations: first, limitations were the number (5) and the selection of participants. The student team in interview 1 (which initiated this study) were participating in a more general study on their learning experiences and use of resources in mathematically-oriented challenge-based courses. The students in interviews 2 and 3 were volunteers with, most likely, a more than average interest in GAI and ChatGPT. This means that the students' schemes and their attitudes towards ChatGPT might not be indicative of those of most mathematics and engineering students. A second limitation is that the questions in interview 1 were not fully focused on answering the research question of this study. As a result, information may be missing from the scheme we presented based on the responses from interview 1.

However, we did elaborate on our original interview questions and expect that the core aspects of this scheme are present. A third limitation is that interviews 1 and 2 were group interviews and that we could not present the schemes of individual students, but that the schemes reflect the content of each interview. Moreover, it is unlikely that the students' processes of instrumental genesis schemes have been completed. Hence, the schemes we presented should not be considered as the final utilization schemes of ChatGPT the students have developed. A fourth limitation is that the schemes we identified were self-reported by the students and their responses, although generally consistent, could not be verified, for example by observations. As a final limitation, we mention that ChatGPT has been developing since the time we

conducted the interviews (e.g., with a connection to a Wolfram Alpha). This implies that the limited and critical interview references to the mathematical help provided by ChatGPT might be considered a snapshot of a still developing situation.

Implications for Practice and Research

In spite of the limitations, the results of this exploratory study show how students, initially without teacher guidance, have started to use ChatGPT, among others as a resource for their studies related to engineering and involving mathematical modelling and programming. With time, schemes of use of ChatGPT gained sophistication (e.g., shown by the responses of student 3). According to the students, this development is likely to continue, also in the light of the development of the tool itself. The students we interviewed showed awareness of the errors made by the software and developed a critical stance; they also showed awareness of the importance to use ChatGPT as a tool for learning and not as a tool to do their work. They used ChatGPT productively in challenge-based projects as well as in more traditional courses, involving mathematical modelling and programming. In particular, in challenge-based education, in which students are likely to need knowledge that is new to them and they need it just-in-time (Gallagher & Savage, 2020), GAI tools may be a fruitful additional resource to foster student agency.

It also became clear from the interviews that the development of a sophisticated scheme of use takes time and critical reflection. Educational institutions might want to monitor, or even guide, this development (which was also suggested by students 3, 4, and 5).

In terms of research, more studies are needed to investigate the use of GAI tools such as ChatGPT by students in mathematics and mathematical modelling, with or without teacher guidance, the schemes they develop regarding its use, and how these use schemes contribute to student learning. This is all the more important as the use of GAI, particularly in mathematics, is likely to be only at its infancy. In addition, and perhaps more importantly, it is necessary to investigate how students, at different educational levels (not only at university), can be successfully supported to use GAI tools.

Appendix A. Interview Questions

Interview 1 was directed to student learning experiences and their use of resources in a challenge-based course, so the questions in that interview were not directly connected to our research question. When ChatGPT was mentioned as a resource, follow-up questions were asked similar to the questions in Table 2.

Interview protocols guided the conduction of the semi-structured interviews 2 and 3. Depending on the student answers additional questions were asked, or the students were asked to elaborate.

Table 2 Sample questions interviews 2 and 3

Interview questions	Remarks
How did you discover ChatGPT, when did you hear about it and when did you start using it? What is your general impression of ChatGPT as a resource?	General introductory questions
In what situations have you used ChatGPT, to solve what kind of activities or tasks?	Addresses the themes: aims and situations
What are the characteristics that you have identified from ChatGPT? What limitations does it have?	Addresses the themes: concepts-in-action and theorems-in-action
How do you know that the information provided by ChatGPT is useful and correct, is it easy to identify?	Mainly addresses the themes: theorems-in-action, rules
After interacting with ChatGPT, what kind of mathematical skills and knowledge have you developed? What other skills (e.g., critical thinking) do you think can be developed by using ChatGPT? Please provide examples	Addresses the contribution to learning, with a focus on mathematics
Do you see ways in which the use of ChatGPT interferes with the development of mathematical and other skills?	Addresses potential disadvantages and limitations of ChatGPT for learning
Please comment on how your use of ChatGPT changed over time	Addresses the development of the students' schemes

Appendix B. Interview Results

Table 3 shows a summary of how we categorized the students' responses in each case in terms of developing schemes of use of ChatGPT. The table entries are summaries or paraphrases of interview quotes and contributions by the different students are indicated (e.g., s1 for student 1). Student 3 (s3) participated both in interview 1 and 2, so his responses in the first two columns of Table 3 are indicative of the development of his scheme.

Table 3 Interview results categorized according to the aspects of a utilization scheme of ChatGPT

Situations

Interview 1	Interview 2	Interview 3
A challenge-based course on modelling crowd flow on a train platform (interview context)	Used in every aspect of their lives, e.g., planning a trip with friends, making a meal plan for the week, career guidance (s3, s4) Study related assignments and concepts (s3, s4) First versions of various writing tasks (e.g., abstract or introduction to research paper) (s3, s4) Career guidance (s3)	Travelling (interesting things to see on the way) Looking for specific words (e.g., "with a particular meaning, rhyming to another word") Programming (generating code), at first longer code, later smaller more specific things Together with friends in Discord call; "messing around," "see if we can break it" Generating questions to use in a quiz game Writing texts for open assignments; in particular the structure is important, not the textual details In the course "applied mathematical algorithms"; aim was to understand the application and what the algorithm did, e.g., the travelling salesman problem; for the programming ChatGPT could be used (encouraged by the teacher) Systems engineering course: writing (part of) a research report (encouraged by the teacher) Writing MATLAB scripts Writing an invitation for a birthday Finding particular (commercial) products in a category (e.g., aggravation blanket); it gives options you can investigate further

Aims

Interview 1	Interview 2	Interview 3
Getting first ideas on Determining a suitable model for crowd flow (s1) Optimizing Python code (Markov chains) (s2) Writing text (topic introduction for report) (s2) Working on visualization for the final report (s3)	Used as a discussion partner to understand concepts better, e.g., in order not to bother professors; to understand assignments. (s4) To create first versions of various writing tasks: emails, small essays, summaries (e.g., for student team), a name for a project, and abstract or introduction to a research paper. (s3, s4) Enhance learning processes; help to understand certain topics. (s4) Be more efficient or better quality of work in the same time. (s3) Investigating options after Bachelor program. (s3)	Exploration, finding possible answer to a question Bringing across the main idea of a text; finding the right words to use in a text Generating Python code Finding suitable MATLAB functions Helping in writing and structuring texts

Table 3 (continued)

Rules-of use

Interview 1	Interview 2	Interview 3
Don't use it for final things (s1, s2, s3) Use the main results of ChatGPT to investigate further (s1, s2)	Instead of copying the (ChatGPT) answer in our work, we enhance our knowledge and use that knowledge to do our work (s4); we figured out that the better way to use ChatGPT is not to let it do your homework, but to help you understand the thing. (s3) If you try to learn a concept, it is important to ask unbiased questions, so that it does not give you biased answers. If you are a little bit biased with your prompts, it will continue in that direction, even if it is wrong. (s4) We always try to ask follow-up questions that confirm it is in the right direction or just hallucinating. (s4) It is bad with mathematics. But it would give you something very general, e.g., a first equation. And it makes mistakes, but you can still get the idea of how to do things from there. (s4) You can say: explain it to me as if I am a kid, and then it will explain it like that and it makes it a lot more clear. (s4) With Google you have to verify that the information is correct. You have to do the same with ChatGPT (assume at first that ChatGPT has it wrong). (s3) It [ChatGPT] can be the middle step before Googling something. E.g. if you don't know how to look it up online. It will for example give you the name of a theorem, but get the theorem wrong. But you can then look up the theorem and give that back to ChatGPT. (s3)	You have to be specific in asking the question Questions on coding: by the time you have asked exactly what you want, you have almost programmed it yourself For school I try to minimize its use, or only ask small questions. Because I am at school to learn things I check if answers are useful (and often it is not, as it makes incorrect assumptions); in programming I check if the code works; and I go through the code to see what exactly it does. Often, I use other internet sources to check what is has said But if you ask for different options, then you have to check less than in case of an answer to a very specific question I usually don't ask: "solve this problem," but "how could I solve this problem." That is good for improving my knowledge Of course. you have to process its information and to start understanding it yourself If I want to write something, it takes me a lot of time to sort out what I want to bring across. And ChatGPT is very good at just making a text, and then I make sure that the result is correct

Concept-in-action

Interview 1	Interview 2	Interview 3
It is like Googling, and getting the main ideas (s1, s2)	It is a new tool and using it may revolutionize many things (compare to the introduction of the calculator). (s3) It can help you to understand things. (s3) You can boost the tool by learning how to prompt it and by using new plugins. (s3) Dangers associated with its use, e.g., security, believing wrong information. (s3) Opportunities associated with its use, e.g., becoming more efficient, so that you can focus on what is important, combining its information with that of other sources (including humans), unlock new ways of learning. (s3)	There appear to be assumptions hidden under the answer given by ChatGPT Similarities and differences with Google

Table 3 (continued)

Theorem-in-action		
Interview 1	**Interview 2**	**Interview 3**
Most details are wrong (s1, s2, s3)	It is good at explaining fundamental concepts in simpler terms. (s4)	It always gives an answer that looks good at first. But is not always correct if you look better
Good to provide (first) ideas and when starting to learn something new ("but it quickly becomes …, not enough") (s1, s2)	It tries to please the user; it will quickly agree with you on more complicated questions (s4); […] if you are a little bit biased with your prompts, then it will continue in that direction, even if it's wrong. (s4)	It answers every question. But some questions cannot be answered, because you don't have enough information
More efficient than Google (depends less on correct search terms; s1, s2)	Similar to Google, you have to verify that the information is correct (as it has been trained on online information and not all of that is correct). (s3, s4)	You can ask it about programming. It gives an answer, but that is probably not what you are looking for
	Although it is not always correct, it still is a kind of guidance and you can make changes yourself (e.g. in programming code). (s4)	If you have specific demands that are not very standard, then it often does not know
	It is pushing you to be critical about the knowledge you receive. (s4); You can think of it as having a discussion with someone that has read every book of mathematics for you to actually understand. Once you understand it then you can be confident that your answer is correct. So instead of seeing it as something that will do the homework for you, you can see it as something that if you tell it to do your homework, it will get it wrong, but it can help you understand things to get it right. (s3); Some people are worried that students will stop learning because they will copy ChatGPT results to their homework; I think if the student does that he will get it all wrong. (s3)	It is often faster than google. And it gives a direct answer to your question
		A school assignment often has a number of requirements that are not easy to give to ChatGPT. And if you could, it would take a lot of time
		In fact, everything you could ask Google, you can also ask ChatGPT
	It seems like someone who is not very intelligent, but has read every book. (s3)	
	It used to be very bad at mathematics; a language model trained on mathematics will be a very inefficient calculator; however if you link it to Wolfram Alpha it will know when to pass a question to Wolfram Alpha and the mathematics will be right most of the time. (s3, s4)	
	Searching for something in Google takes much more time; ChatGPT gives you an answer directly related to your prompt; unlike Google, it generates content related to the things you give it. (s3)	
	With the new browsing mode, I don't have to Google things before I give them to ChatGPT as it can Google things itself. (s3)	
	It allows you to do things more efficiently or in the same amount of time at higher quality. (s3)	

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Data Availability The raw interview data on which this study is based are subject to Privacy Laws and Informed Consent given by the participants. Therefore, the data are not publicly available. Access to (part of) the data can be given upon reasonable request to the authors.

Declarations

Ethics Approval Permission to conduct the study was obtained by Ethical Review Board of the Eindhoven University of Technology under reference ERB2021ESOE9 as part of the project "Student learning experiences in challenge-based education: the case/s of applied mathematics and physics." The participants to the study gave their informed consent to be interviewed and to process the interview data.

Competing Interests The authors declare no competing interests.

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